

Robust Decentralized Abstractions for Multiple Mobile Manipulators

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Abstract—This paper addresses the problem of decentralized abstractions for multiple mobile manipulators with 2nd order dynamics. In particular, we propose decentralized controllers for the navigation of each agent among predefined regions of interest in the workspace, while guaranteeing at the same time inter-agent collision avoidance and connectivity maintenance for a subset of initially connected agents. In that way, the motion of the coupled multi-agent system is abstracted into multiple finite transition systems for each agent, which are then suitable for the application of temporal logic-based high level plans. The proposed methodology is decentralized, since each agent uses local information based on limited sensing capabilities. Finally, simulation studies verify the validity of the approach.

I. INTRODUCTION

Multi-agent systems have gained a significant amount of attention in the last decades, due to the several advantages they yield with respect to single-agent setups. A recent direction in the multi-agent control and robotics field is the use of temporal logic languages for motion and/or action planning, since they provide a fully-automated correct-by-design controller synthesis approach for autonomous robots. Temporal logics, such as linear temporal logic (LTL), computation tree logic (CTL) or metric-interval temporal logic (MITL), provide formal high-level languages that can describe planning objectives more complex than the usual navigation techniques. The task specification is given as a temporal logic formula with respect to a discretized abstraction of the robot motion modeled as a finite transition system, and then, a high-level discrete plan is found by off-the-shelf model-checking algorithms, given the finite transition system and the task specification [1].

There exists a wide variety of works that employ temporal logic languages for multi-agent systems, e.g., [2]–[9]. The discretization of a multi-agent system to an abstracted finite transition system necessitates the design of appropriate continuous-time controllers for the transition of the agents among the states of the transition system [1]. Most works in the related literature, however, including the aforementioned ones, either assume that there *exist* such continuous controllers or adopt single- and double-integrator models, ignoring the actual dynamics of the agents. Discretized abstractions, including design of the discrete state space and/or continuous-time controllers, have been considered

in [10]–[14] for general systems and [15], [16] for multi-agent systems. Another important issue concerning multi-agent abstractions that has not been addressed in the related literature, is the collision avoidance between the robotic agents, which, unlike the unrealistic point-mass assumption that is considered in many works, can be more appropriately approximated by unions of rigid bodies.

This work addresses the problem of decentralized abstractions for a team of mobile robotic manipulators, represented by a union of 3D ellipsoids, among predefined regions of interest in the workspace. Mobile manipulators consist of a mobile base and a robotic arm, which makes them suitable for performing actions around a workspace (e.g., transportation of objects). In [17] the authors consider the navigation of two mobile manipulators grasping an object, based on 3D ellipsoids, whereas [18] deals with general-shape multi-agent navigation, both based on point-world transformations. Navigation of ellipsoidal agents while incorporating collision-avoidance properties was also studied in [19] for single-integrator dynamics, by transforming the ellipsoids to spheres. In our previous work [20], we addressed a hybrid control framework for the navigation of mobile manipulators and their interaction with objects in a given workspace, proposing, however, a centralized solution.

In this work, we design robust continuous-time controllers for the navigation of the agents among the regions of interest. The proposed methodology is decentralized, since each agent uses only local information based on limited sensing capabilities. Moreover, we guarantee (i) inter-agent collision avoidance by introducing a novel transformation-free ellipsoid-based strategy, (ii) connectivity maintenance for a subset of the initially connected agents, which might be important for potential cooperative tasks, and (iii) kinematic singularity avoidance of the robotic agents. The overall approach is based on appropriately chosen potential fields, that are provably free of undesired local minima.

The rest of the paper is organized as follows. Section II provides necessary notation and preliminary background and Section III describes the tackled problem. The main results are given in Section IV and Section V presents simulation results. Finally, VI concludes the paper.

II. PRELIMINARIES

A. Notation

The set of positive integers is denoted as \mathbb{N} whereas the real and complex n -coordinate spaces, with $n \in \mathbb{N}$, are denoted as \mathbb{R}^n and \mathbb{C}^n , respectively; $\mathbb{R}_{\geq 0}^n$, $\mathbb{R}_{> 0}^n$, $\mathbb{R}_{\leq 0}^n$ and $\mathbb{R}_{< 0}^n$ are the sets of real n -vectors with all elements nonnegative, positive, nonpositive, and negative, respectively. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$.

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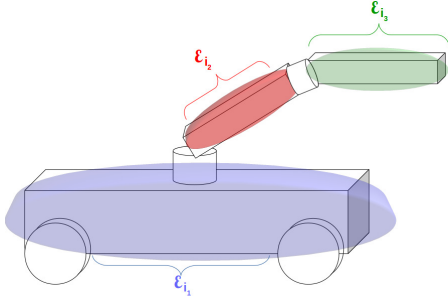


Fig. 1: An agent that consists of $\ell_i = 3$ rigid links.

Given a scalar function $y : \mathbb{R}^n \rightarrow \mathbb{R}$ and a vector $x \in \mathbb{R}^n$, we use the notation $\nabla_x y(x) = [\frac{\partial y}{\partial x_1}, \dots, \frac{\partial y}{\partial x_n}]^T \in \mathbb{R}^n$. Define by $I_n \in \mathbb{R}^{n \times n}$, $0_{m \times n} \in \mathbb{R}^{m \times n}$, the identity matrix and the $m \times n$ matrix with all entries zeros, respectively; $\mathcal{B}_{c,r} = \{x \in \mathbb{R}^3 : \|x - c\| \leq r\}$ is the 3D sphere of center $c \in \mathbb{R}^3$ and radius $r \in \mathbb{R}_{\geq 0}$. The boundary of a set A is denoted as ∂A and its interior as $\mathring{A} = A \setminus \partial A$. The vector connecting the origins of coordinate frames $\{A\}$ and $\{B\}$ expressed in frame $\{C\}$ coordinates in 3D space is denoted as $p_{B/A}^C \in \mathbb{R}^3$. For notational brevity, when a coordinate frame corresponds to an inertial frame of reference $\{I\}$, we will omit its explicit notation (e.g., $p_B = p_{B/I}^I$, $\omega_B = \omega_{B/I}^I$). All vector and matrix differentiations are derived with respect to an inertial frame $\{I\}$, unless otherwise stated.

B. Cubic Equations and Ellipsoid Collision

Proposition 1: Consider the cubic equation $f(\lambda) = c_3\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0$ with $c_\ell \in \mathbb{R}, \forall \ell \in \{0, \dots, 3\}$ and roots $(\lambda_1, \lambda_2, \lambda_3) \in \mathbb{C}^3$, with $f(\lambda_1) = f(\lambda_2) = f(\lambda_3) = 0$. Then, given its discriminant $\Delta = (c_3)^4 \prod_{i \in \{1,2\}} \prod_{j \in \{i+1, \dots, 3\}} (\lambda_i - \lambda_j)^2$, the following hold:

- (i) $\Delta = 0 \Leftrightarrow \exists i, j \in \{1, 2, 3\}$, with $i \neq j$, such that $\lambda_i = \lambda_j$, i.e., at least two roots are equal,
- (ii) $\Delta > 0 \Leftrightarrow \lambda_i \in \mathbb{R}, \forall i \in \{1, 2, 3\}$, and $\lambda_i \neq \lambda_j, \forall i, j \in \{1, 2, 3\}$, with $i \neq j$, i.e., all roots are real and distinct.

Proposition 2: [21] Consider two planar ellipsoids $\mathcal{A} = \{z \in \mathbb{R}^3 \text{ s.t. } z^T A(t) z \leq 0\}$, $\mathcal{B} = \{z \in \mathbb{R}^3 \text{ s.t. } z^T B(t) z \leq 0\}$, with $z = [p^T, 1]^T$ being the homogeneous coordinates of $p \in \mathbb{R}^2$, and $A, B : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{3 \times 3}$ terms that describe their motion in 2D space. Given their characteristic polynomial $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(\lambda) = \det(\lambda A - B)$, which has degree 3, the following hold:

- (i) $\exists \lambda^* \in \mathbb{R}_{>0}$ s.t. $f(\lambda^*) = 0$, i.e., the polynomial $f(\lambda)$ has always one positive real root,
- (ii) $\mathcal{A} \cap \mathcal{B} = \emptyset \Leftrightarrow \exists \lambda_1^*, \lambda_2^* \in \mathbb{R}_{<0}$, with $\lambda_1^* \neq \lambda_2^*$, and $f(\lambda_1^*) = f(\lambda_2^*) = 0$, i.e., \mathcal{A} and \mathcal{B} are disjoint if and only if the characteristic equation $f(\lambda) = 0$ has two distinct negative roots.
- (iii) $\mathcal{A} \cap \mathcal{B} \neq \emptyset$ and $\mathring{\mathcal{A}} \cap \mathring{\mathcal{B}} = \emptyset \Leftrightarrow \exists \lambda_1^*, \lambda_2^* \in \mathbb{R}_{<0}$, with $\lambda_1^* = \lambda_2^*$, and $f(\lambda_1^*) = f(\lambda_2^*) = 0$, i.e., \mathcal{A} and \mathcal{B} touch externally if and only if the characteristic equation $f(\lambda) = 0$ has a negative double root.

III. PROBLEM FORMULATION

Consider $N \in \mathbb{N}$ fully actuated agents with $\mathcal{V} := \{1, \dots, N\}$, $N \geq 2$, composed by a robotic arm mounted

on an omnidirectional mobile base, operating in a static workspace \mathcal{W} that is bounded by a large sphere in 3D space, i.e. $\mathcal{W} = \mathcal{B}_{p_0, r_0} = \{p \in \mathbb{R}^3 \text{ s.t. } \|p - p_0\| \leq r_0\}$, where $p_0 \in \mathbb{R}^3$ is the center of \mathcal{W} , and $r_0 \in \mathbb{R}_{\geq 0}$ is its radius. Without loss of generality, we consider that $p_0 = 0_{3 \times 1}$, corresponding to an inertial frame $\{I\}$. Within \mathcal{W} there exist K disjoint spheres around points of interest, which are described by $\pi_k = \mathcal{B}_{p_k, r_k} = \{p \in \mathbb{R}^3 \text{ s.t. } \|p - p_k\| \leq r_k\}$, $k \in \mathcal{K} := \{1, \dots, K\}$, where $p_k \in \mathbb{R}^3$ and $r_k \in \mathbb{R}_{>0}$ are the center and radius of the k th region, respectively. The regions of interest can be equivalently described by $\pi_k = \{z \in \mathbb{R}^4 \text{ s.t. } z^T T_{\pi_k} z \leq 0\}$, where $z = [p^T, 1]^T$ is the vector of homogeneous coordinates of $p \in \mathbb{R}^3$, and

$$T_{\pi_k} = \begin{bmatrix} I_3 & p_k \\ 0_{3 \times 1}^T & -r_k^2 \end{bmatrix}, \forall k \in \mathcal{K}. \quad (1)$$

The dynamic model of each agent is given by the second-order Lagrangian dynamics:

$$M_i(q_i) \ddot{q}_i + N_i(q_i, \dot{q}_i) \dot{q}_i + g_i(q_i) + f_i(q_i, \dot{q}_i) = \tau_i, \quad (2)$$

$\forall i \in \mathcal{V}$, where $q_i : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{n_i}$ is the vector of generalized coordinates (e.g., pose of mobile base and joint coordinates of the arms), $M_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_i \times n_i}$ is the positive definite inertia matrix, $N_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_i \times n_i}$ is the Coriolis matrix, $g_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_i}$ is the gravity vector, $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{n_i}$ is a term representing friction and modeling uncertainties and $\tau_i \in \mathbb{R}^{n_i}$ is the vector of joint torques, representing the control inputs. Without loss of generality, we assume that $n_i = n \in \mathbb{N}, \forall i \in \mathcal{V}$. In addition, we denote as $\{B_i\}$ the frame of the mobile base of agent i and $p_{B_i} : \mathbb{R}^n \rightarrow \mathbb{R}^3$ its inertial position. Moreover, the matrix $\dot{M}_i - 2N_i$ is skew-symmetric [22], and we further make the following assumption:

Assumption 1: The term $f_i(q_i, \dot{q}_i)$ satisfies $f(q_i, 0_{n \times 1}) = 0_{n \times 1}, \forall q_i \in \mathbb{R}^n$, and there exist positive constants c_i such that $\|f_i(q_i, \dot{q}_i)\| \leq c_i \|q_i\| \|\dot{q}_i\|, \forall (q_i, \dot{q}_i) \in \mathbb{R}^n \times \mathbb{R}^n, i \in \mathcal{V}$.

We consider that each agent is composed by ℓ_i rigid links (see Fig. 1) with $\mathcal{L}_i = \{1, \dots, \ell_i\}$ the corresponding index set. Each link of agent i is approximated by the ellipsoid set [21] $\mathcal{E}_{i_m}(q_i) = \{z \in \mathbb{R}^4 \text{ s.t. } z^T E_{i_m}(q_i) z \leq 0\}$; $z = [p^T, 1]^T$ is the homogeneous coordinates of $p \in \mathbb{R}^3$, and $E_{i_m} : \mathbb{R}^n \rightarrow \mathbb{R}^{4 \times 4}$ is defined as $E_{i_m}(q_i) = T_{i_m}^{-T}(q_i) \hat{E}_{i_m} T_{i_m}^{-1}(q_i)$, where $\hat{E}_{i_m} = \text{diag}\{a_{i_m}^{-2}, b_{i_m}^{-2}, c_{i_m}^{-2}, -1\}$ corresponds to the positive lengths $a_{i_m}, b_{i_m}, c_{i_m}$ of the principal axes of the ellipsoid, and $T_{i_m} : \mathbb{R}^n \rightarrow \mathbb{R}^{4 \times 4}$ is the transformation matrix for the coordinate frame $\{i_m\}$ placed at the center of mass of the m -th link of agent i , aligned with the principal axes of \mathcal{E}_{i_m} :

$$T_{i_m}(q_i) = \begin{bmatrix} R_{i_m}(q_i) & p_{i_m}(q_i) \\ 0_{3 \times 1}^T & 1 \end{bmatrix},$$

with $R_{i_m} : \mathbb{R}^n \rightarrow \mathbb{R}^{3 \times 3}$ being the rotation matrix of the center of mass of the link, $\forall m \in \mathcal{L}_i, i \in \mathcal{V}$. For an ellipsoid $\mathcal{E}_{i_m}, i \in \mathcal{V}, m \in \mathcal{L}_i$, we denote as $\mathcal{E}_{i_m}^{xy}, \mathcal{E}_{i_m}^{xz}, \mathcal{E}_{i_m}^{yz}$ its projections on the planes x - y , x - z and y - z , respectively, with corresponding matrix terms $E_{i_m}^{xy}, E_{i_m}^{xz}, E_{i_m}^{yz}$. Note that

the following holds for two different ellipsoids \mathcal{E}_{i_m} and \mathcal{E}_{j_l} :

$$\begin{aligned} \mathcal{E}_{i_m}(q_i) \cap \mathcal{E}_{j_l}(q_j) &\neq \emptyset \wedge \hat{\mathcal{E}}_{i_m}(q_i) \cap \hat{\mathcal{E}}_{j_l}(q_j) = \emptyset \Leftrightarrow \\ \mathcal{E}_{i_m}^s(q_i) \cap \mathcal{E}_{j_l}^s(q_j) &\neq \emptyset \wedge \hat{\mathcal{E}}_{i_m}^s(q_i) \cap \hat{\mathcal{E}}_{j_l}^s(q_j) = \emptyset, \end{aligned}$$

$\forall s \in \{xy, xz, yz\}$, i.e., in order for $\mathcal{E}_{i_m}, \mathcal{E}_{j_l}$ to collide (touch externally), all their projections on the three planes must also collide. Therefore, a sufficient condition for \mathcal{E}_{i_m} and \mathcal{E}_{j_l} not to collide is $\mathcal{E}_{i_m}^s(q_i) \cap \mathcal{E}_{j_l}^s(q_j) = \emptyset$, for some $s \in \{xy, xz, yz\}$. In view of Proposition 2, that means that the characteristic equations $f_{i_m, j_l}^s(\lambda) := \det(\lambda \mathcal{E}_{i_m}^s(q_i) - \mathcal{E}_{j_l}^s(q_j)) = 0$ must always have one positive real root and two negative distinct roots for at least one $s \in \{xy, xz, yz\}$. Hence, by denoting the discriminant of $f_{i_m, j_l}^s(\lambda) = 0$ as Δ_{i_m, j_l}^s , Proposition 1 suggests that Δ_{i_m, j_l}^s must remain always positive for at least one $s \in \{xy, xz, yz\}$, since a collision would imply $\Delta_{i_m, j_l}^s = 0$, $\forall s \in \{xy, xz, yz\}$. Therefore, by defining the function $\delta : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}$ as:

$$\delta(x) = \begin{cases} \phi_\delta(x), & x > 0, \\ 0, & x \leq 0, \end{cases} \quad (3)$$

where ϕ_δ is an appropriate polynomial that ensures that $\delta(x)$ is twice continuously differentiable everywhere (e.g. $\phi_\delta(x) = x^3$), we can conclude that a sufficient condition for \mathcal{E}_{i_m} and \mathcal{E}_{j_l} not to collide is $\delta(\Delta_{i_m, j_l}^{xy}) + \delta(\Delta_{i_m, j_l}^{xz}) + \delta(\Delta_{i_m, j_l}^{yz}) > 0$, since a collision would result in $\Delta_{i_m, j_l}^s = 0 \Leftrightarrow \delta(\Delta_{i_m, j_l}^s) = 0$, $\forall s \in \{xy, xz, yz\}$.

Next, we define the constant \bar{d}_{B_i} , which is the maximum distance of the base to a point in the agent's volume over all possible configurations, i.e. $\bar{d}_{B_i} = \sup_{q_i \in \mathbb{R}^n} \{\|p_{B_i} - p_i(q_i)\|\}$, $p_i \in \bigcup_{m \in \mathcal{L}_i} \mathcal{E}_{i_m}(q_i)$. We also denote $\bar{d}_B = [\bar{d}_{B_1}, \dots, \bar{d}_{B_N}]^\top \in \mathbb{R}_{\geq 0}^N$.

Moreover, we consider that each agent has a sensor located at the center of its mobile base p_{B_i} with a sensing radius $d_{\text{con}_i} \geq 2 \max_{i \in \mathcal{V}} \{\bar{d}_{B_i}\} + \varepsilon_d$, where ε_d is an arbitrarily small positive constant. Hence, each agent has the sensing sphere $\mathcal{D}_i(q_i) = \{p \in \mathbb{R}^3 \text{ s.t. } \|p - p_{B_i}\| \leq d_{\text{con}_i}\}$ and its neighborhood set at each time instant is defined as $\mathcal{N}_i(t) = \{j \in \mathcal{V} \setminus \{i\} \text{ s.t. } \|p_{B_i} - p_{B_j}\| \leq d_{\text{con}_i}\}$.

As mentioned in Section I, we are interested in defining transition systems for the motion of the agents in the workspace in order to be able to assign complex high level goals through logic formulas. Moreover, since many applications necessitate the cooperation of the agents in order to execute some task (e.g. transport an object), we consider that a nonempty subset $\tilde{\mathcal{N}}_i \subseteq \mathcal{N}_i(0)$, $i \in \mathcal{V}$, of the initial neighbors of the agents must stay connected through their motion in the workspace. In addition, it follows that the transition system of each agent must contain information regarding the current position of its neighbors. The problem in hand is equivalent to designing decentralized control laws τ_i , $i \in \mathcal{V}$, for the appropriate transitions of the agents among the predefined regions of interest in the workspace.

Next, we provide the following necessary definitions.

Definition 1: An agent $i \in \mathcal{V}$ is in region $k \in \mathcal{K}$ at a configuration $q_i \in \mathbb{R}^n$, denoted as $\mathcal{A}_i(q_i) \in \pi_k$, if and only if $\|p_{i_m}(q_i) - p_k\| \leq r_k - \max\{\alpha_{i_m}, \beta_{i_m}, c_{i_m}\}$, $\forall m \in \mathcal{L}_i \Leftrightarrow \|p_{B_i}(q_i) - p_k\| \leq r_k - \bar{d}_{B_i}$.

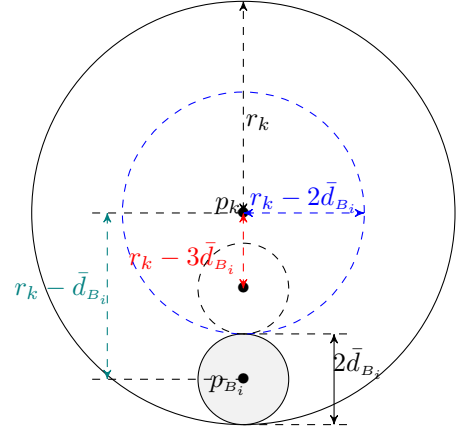


Fig. 2: An example of a region π_k and an agent i , with $r_k = 7$, $\bar{d}_{B_i} = 1.5$, which we consider to be the maximum \bar{d}_{B_i} , and $\bar{r}(N, \bar{d}_B) = \tilde{r}(N, \bar{d}_B) = 3$. It holds then that $r_k - 3\bar{d}_{B_i} = 2.5 > 1$, as shown in (4). The analysis in Section IV shows that the agent will converge to the subset defined by the blue circle, with radius $r_k - 2\bar{d}_{B_i} = 4 \geq \bar{r}(N, \bar{d}_B) \geq \tilde{r}(N, \bar{d}_B)$, (i.e. the distance from p_{B_i} to p_k will become less than $r_k - 3\bar{d}_{B_i}$). Hence, there exists a finite positive time instant t_{f_i} , that the agent will be in π_k (i.e., the distance from p_{B_i} to p_k will be less than $r_k - \bar{d}_{B_i}$ at t_{f_i}).

Definition 2: Agents $i, j \in \mathcal{V}$, with $i \neq j$, are in *collision-free* configurations $q_i, q_j \in \mathbb{R}^n$, denoted as $\mathcal{A}_i(q_i) \neq \mathcal{A}_j(q_j)$, if and only if $\mathcal{E}_{i_m}(q_i) \cap \mathcal{E}_{j_l}(q_j) = \emptyset$, $\forall m \in \mathcal{L}_i, l \in \mathcal{L}_j$.

Given the aforementioned discussion, we make the following assumptions regarding the agents and the validity of the workspace:

Assumption 2: Agent $i \in \mathcal{V}$ at time t has feedback of the neighboring coordinates and rates q_j, \dot{q}_j , $\forall j \in \mathcal{N}_i(t)$.

Assumption 3: Consider the function $\tilde{r} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathbb{R}_{\geq 0}$ that computes the minimum radius of a region such that it can contain all the agents at the same time, i.e., $\tilde{r}(N, \bar{d}_B) = \min_{r \in \mathbb{R}_{\geq 0}} \{r \in \mathbb{R}_{\geq 0} \text{ s.t. } \mathcal{A}_i(q_i) \in \pi, \mathcal{A}_i(q_i) \neq \mathcal{A}_j(q_j), \forall i, j \in \mathcal{V}, i \neq j\}$ where $\pi := \{p \in \mathbb{R}^3 \text{ s.t. } \|p\| \leq r\}$. Consider also the function $\tilde{r} : \mathbb{N} \times \mathbb{R}^N \rightarrow \mathbb{R}_{\geq 0}$, where

$$\tilde{r}(N, \bar{d}_B) = \max\{\tilde{r}(N, \bar{d}_B), 1 + \max_{i \in \mathcal{V}} \{\bar{d}_{B_i}\}\}. \quad (4)$$

Then it holds that

- (i) $r_k \geq 2 \max_{i \in \mathcal{V}} \{\bar{d}_{B_i}\} + \tilde{r}(N, \bar{d}_B)$, $\forall k \in \mathcal{K}$,
- (ii) $\|p_k - p_{k'}\| \geq \max_{i \in \mathcal{V}} \{2\bar{d}_{B_i}\} + r_k + r_{k'} + \varepsilon_p$, $\forall k, k' \in \mathcal{K}, k \neq k'$,

for an arbitrarily small positive constant ε_p .

Statement (i) of Assumption 3 is needed for cases where more than one agents need to navigate to the same region of interest; Intuitively, it states that, if needed, all agents can be confined in a subset of a region with radius greater both than $\tilde{r}(N, \bar{d}_B)$ and $1 + \max_{i \in \mathcal{V}} \{\bar{d}_{B_i}\}$. The latter, which is important for the subsequent analysis, can be interpreted as the regions being sufficiently large such that the agents can navigate there in finite time, as will be shown later. Moreover, statement (ii) suggests that the regions of interest are sufficiently far from each other so that each agent is able to navigate among them without intersecting them, which is a necessary assumption for the feasibility of the problem. Note that, intuitively, assumption 3 implies that the workspace \mathcal{W}

has to be sufficiently large. Note that it follows from (i) and (4) that

$$\begin{aligned} r_k - 3\bar{d}_{B_i} &\geq r_k - 3\max_{i \in \mathcal{V}}\{\bar{d}_{B_i}\} \\ &\geq \tilde{r}(N, \bar{d}_B) - \max_{i \in \mathcal{V}}\{\bar{d}_{B_i}\} \geq 1, \end{aligned} \quad (5)$$

$\forall k \in \mathcal{K}, i \in \mathcal{V}$, which will be needed in the following analysis. An illustration of a region of interest that incorporates Assumption 3 is given in Fig. 2. Next, in order to proceed, we need the following definition.

Definition 3: Assume that $\mathcal{A}_i(q_i(t_0)) \in \pi_k, i \in \mathcal{V}$, for some $t_0 \in \mathbb{R}_{\geq 0}, k \in \mathcal{K}$, with $\mathcal{A}_i(q_i(t_0)) \neq \mathcal{A}_j(q_j(t_0)), \forall j \in \mathcal{V} \setminus \{i\}$. There exists a transition for agent i between π_k and $\pi_{k'}, k' \in \mathcal{K}$, denoted as $(\pi_k, t_0) \xrightarrow{i} (\pi_{k'}, t_f)$, if and only if there exists a finite time $t_f \geq t_0$, such that $\mathcal{A}_i(q_i(t_f)) \in \pi_{k'}$ and $\mathcal{A}_i(q_i(t)) \neq \mathcal{A}_j(q_j(t)), \mathcal{E}_{i_m}(q_i(t_0)) \cap \mathcal{E}_{i_\ell}(q_i(t_0)), \mathcal{E}_{i_m}(q_i(t)) \cap \pi_z = \emptyset, \forall m, \ell \in \mathcal{L}_i, m \neq \ell, j \in \mathcal{V} \setminus \{i\}, z \in \mathcal{K} \setminus \{k, k'\}, t \in [t_0, t_f]$.

Given the aforementioned definitions, the treated problem is the design of decentralized control laws for the transitions of the agents between two regions of interest in the workspace, while preventing collisions of the agents with each other, the workspace boundary, and the remaining regions of interest. More specifically, we aim to design a finite transition system for each agent of the form [1]

$$\mathcal{T}_i = (\Pi, \Pi_{i,0}, \xrightarrow{i}, \mathcal{AP}_i, \mathcal{L}_i, \mathcal{F}_i), \quad (6)$$

where $\Pi = \{\pi_1, \dots, \pi_K\}$ is the set of regions of interest that the agents can be at, according to Def. 1, $\Pi_{i,0} \subseteq \Pi$ is a set of initial regions that each agent can start from, $\xrightarrow{i} \subset (\Pi \times \mathbb{R}_{\geq 0})^2$ is the transition relation of Def. 3, \mathcal{AP}_i is a set of given atomic propositions, represented as boolean variables, that hold in the regions of interest, $\mathcal{L}_i : \Pi \rightarrow 2^{\mathcal{AP}_i}$ is a labeling function, and $\mathcal{F}_i : \Pi \rightarrow \Pi^{|\tilde{\mathcal{N}}_i|}$ is a function that maps the region that agent i occupies to the regions the initial neighbors $\tilde{\mathcal{N}}_i$ of agent i are at. Therefore, the treated problem is the design of bounded controllers τ_i for the establishment of the transitions \xrightarrow{i} . Moreover, as discussed before, the control protocol should also guarantee the connectivity maintenance of a subset of the initial neighbors $\tilde{\mathcal{N}}_i, \forall i \in \mathcal{V}$. Another desired property important in applications involving robotic manipulators, is the nonsingularity of the Jacobian matrix $J_i : \mathbb{R}^n \rightarrow \mathbb{R}^{6 \times n}$, that transforms the generalized coordinate rates of agent $i \in \mathcal{V}$ to generalized velocities [22]. That is, the set $\mathbb{S}_i = \{q_i \in \mathbb{R}^n \text{ s.t. } \det(J_i(q_i)) = 0\}$ should be avoided, $\forall i \in \mathcal{V}$. Formally, we define the problem treated in this paper as follows:

Problem 1: Consider N mobile manipulators with dynamics (2) and K regions of interest $\pi_k, k \in \mathcal{K}$, with $\dot{q}_i(t_0) < \infty, \mathcal{A}_i(q_i(t_0)) \in \pi_{k_i}, k_i \in \mathcal{K}, \forall i \in \mathcal{V}$ and $\mathcal{A}_i(q_i(t_0)) \neq \mathcal{A}_j(q_j(t_0)), \mathcal{E}_{i_m}(q_i(t_0)) \cap \mathcal{E}_{i_\ell}(q_i(t_0)) = \emptyset, \forall i, j \in \mathcal{V}, i \neq j, m, \ell \in \mathcal{L}_i, m \neq \ell$. Given nonempty subsets of the initial edge sets $\tilde{\mathcal{N}}_i \subseteq \tilde{\mathcal{N}}_i(0) \subseteq \mathcal{V}, \forall i \in \mathcal{V}$, the fact that $\det(J_i(q_i(t_0))) \neq 0, \forall i \in \mathcal{V}$, as well as the indices $k'_i \in \mathcal{K}, i \in \mathcal{V}$, such that $\|p_{k'_i} - p_{k'_j}\| + r_{k'_i} + r_{k'_j} \leq d_{\text{con}_i}, \forall j \in \tilde{\mathcal{N}}_i, i \in \mathcal{V}$, design decentralized controllers τ_i such that, for all $i \in \mathcal{V}$:

- 1) $(\pi_{k_i}, t_0) \xrightarrow{i} (\pi_{k'_i}, t_{f_i})$, for some $t_{f_i} \geq t_0$,
- 2) $r_0 - (\|p_{B_i}(t)\| + \bar{d}_{B_i}) > 0, \forall t \in [t_0, t_{f_i}]$,
- 3) $j_i^* \in \mathcal{N}_i(t), \forall j_i^* \in \tilde{\mathcal{N}}_i, t \in [t_0, t_{f_i}]$,
- 4) $q_i(t) \in \mathbb{R}^n \setminus \mathbb{S}_i, \forall t \in [t_0, t_{f_i}]$.

The aforementioned specifications concern 1) the agent transitions according to Def. 3, 2) the confinement of the agents in \mathcal{W} , 3) the connectivity maintenance between a subset of initially connected agents and 4) the agent singularity avoidance. Moreover, the fact that the initial edge sets $\tilde{\mathcal{N}}_i$ are nonempty implies that the sensing radius of each agent i covers the regions π_{k_j} of the agents in the neighboring set $\tilde{\mathcal{N}}_i$. Similarly, the condition $\|p_{k'_i} - p_{k'_j}\| + r_{k'_i} + r_{k'_j} \leq d_{\text{con}_i}, \forall j \in \tilde{\mathcal{N}}_i$, is a feasibility condition for the goal regions, since otherwise it would be impossible for two initially connected agents to stay connected. Intuitively, the sensing radii d_{con_i} should be large enough to allow transitions of the multi-agent system to the entire workspace.

IV. MAIN RESULTS

A. Continuous Control Design

To solve Problem 1, we first define a *decentralized potential function* $\varphi_i : \mathbb{R}^n \rightarrow [0, 1]$ for each agent $i \in \mathcal{V}$, as:

$$\varphi_i(q) = \frac{\gamma_i(q_i)}{\left(\gamma_i^{k_i}(q_i) + \beta_i(q)\right)^{\frac{1}{k_i}}}, \quad (7)$$

where $\gamma_i : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ is the *goal function* that should vanish when $\mathcal{A}_i(q_i) \in \pi_{k'_i}$, $\beta_i : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ is a bounded *obstacle function* that encodes inter-agent collisions, collisions between the agents and the obstacle boundary/undesired regions of interest, connectivity losses between initially connected agents and singularities of the Jacobian matrix $J_i(q_i)$; β_i vanishes when one or more of the above situation occurs; $k_i \in \mathbb{N}$ is a tuning parameter satisfying $k_i > 1, \forall i \in \mathcal{V}$. Note that the goal here is to eventually drive φ_i to zero, while keeping it strictly lower than one, since $\varphi_i < 1 \Leftrightarrow \beta_i > 0$, and hence, we avoid collisions, connectivity breaks, and Jacobian singularities. The potential function (7) is motivated by [23], where it was first introduced as a provably correct-by-construction navigation function. In this work, however, (7) is not required to have the necessary properties by design, as will be shown subsequently.

1) γ_i - *Goal Function*: Function γ_i encodes the control objective of agent i , i.e., reach the region of interest $\pi_{k'_i}$. Hence, we define $\gamma_i : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ as

$$\gamma_i(q_i) = \|p_{B_i}(q_i) - p_{k'_i}\|^2, \quad (8)$$

that represents the distance of the i th agent's base to the center of the goal region. Note that it is not necessary to guarantee that γ_i reaches its global minimum ($p_{B_i} = p_{k'_i}$), but instead, the more relaxed condition $\gamma_i(q_i) \leq (r_{k'_i} - \bar{d}_{B_i})^2$, which is equivalent to $\mathcal{A}_i(q_i) \in \pi_{k'_i}, \forall i \in \mathcal{V}$.

2) β_i - *Collision/Connectivity/Singularity Function*: The function β_i encodes all inter-agent collisions, collisions with the boundary of the workspace and the undesired regions of interest, connectivity between initially connected agents and singularities of the Jacobian matrix $J_i(q_i), \forall i \in \mathcal{V}$.

Consider the function $\Delta_{i_m, j_l} : \mathbb{R}^{2n} \rightarrow \mathbb{R}_{\geq 0}$, with $\Delta_{i_m, j_l}(q_i, q_j) = \delta(\Delta_{i_m, j_l}^{xy}(q_i, q_j)) + \delta(\Delta_{i_m, j_l}^{xz}(q_i, q_j)) + \delta(\Delta_{i_m, j_l}^{yz}(q_i, q_j))$, where $\Delta_{i_m, j_l}^s : \mathbb{R}^{2n} \rightarrow \mathbb{R}_{\geq 0}$ is the discriminant of the cubic equation $\det\{\lambda E_{i_m}^s(q_i) - E_{j_l}^s(q_j)\} = 0, \forall s \in \{xy, xz, yz\}$, for two given ellipsoids \mathcal{E}_{i_m} and $\mathcal{E}_{j_l}, m \in \mathcal{L}_i, l \in \mathcal{L}_j, i, j \in \mathcal{V}$, and δ as defined in (3). As discussed in Section III, a sufficient condition for the ellipsoids \mathcal{E}_{i_m} and \mathcal{E}_{j_l} not to collide, is $\Delta_{i_m, j_l}(q_i(t), q_j(t)) > 0, \forall t \in \mathbb{R}_{\geq 0}$.

Additionally, we define the greatest lower bound of the Δ_{i_m, j_l} when the point p_{j_l} is on the boundary of the sensing radius $\partial D_i(q_i)$ of agent i , as $\tilde{\Delta}_{i_m, j_l} = \inf_{(q_i, q_j) \in \mathbb{R}^{2n}} \{\Delta_{i_m, j_l}(q_i, q_j)\} \text{ s.t. } \|p_{B_i}(q_i) - p_{j_l}(q_j)\| = d_{\text{con}_i}, \forall m \in \mathcal{L}_i, l \in \mathcal{L}_j, i, j \in \mathcal{V}$. Since $d_{\text{con}_i} > 2 \max_{i \in \mathcal{V}} \{\bar{d}_{B_i}\} + \varepsilon_d$, it follows that there exists a positive constant ε_Δ such that $\tilde{\Delta}_{i_m, j_l} \geq \varepsilon_\Delta > 0, \forall m \in \mathcal{L}_i, l \in \mathcal{L}_j, i, j \in \mathcal{V}, i \neq j$.

Moreover, we define the function $\Delta_{i_m, \pi_k} : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$, with $\Delta_{i_m, \pi_k}(q_i) = \delta(\Delta_{i_m, \pi_k}^{xy}(q_i)) + \delta(\Delta_{i_m, \pi_k}^{xz}(q_i)) + \delta(\Delta_{i_m, \pi_k}^{yz}(q_i))$, where $\Delta_{i_m, \pi_k}^s : \mathbb{R}^n \rightarrow \mathbb{R}$ is the discriminant of the cubic equation $\det(\lambda E_{i_m}^s(q_i) - T_{\pi_k}^s)$, with $T_{\pi_k}^s$ the projected version of T_{π_k} in (1), $s \in \{xy, xz, yz\}$, and δ as given in (3). A sufficient condition for \mathcal{E}_{i_m} and region $\pi_k, k \in \mathcal{K}$ not to collide is $\Delta_{i_m, \pi_k}(q_i(t)) > 0, \forall t \in \mathbb{R}_{\geq 0}, m \in \mathcal{L}_i, i \in \mathcal{V}$.

We further define the function $\eta_{ij, c} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$, with $\eta_{ij, c}(q_i, q_j) = d_{\text{con}_i}^2 - \|p_{B_i}(q_i) - p_{B_j}(q_j)\|^2$, and the distance functions $\beta_{i_m, j_l} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}, \beta_{ij, c} : \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}, \beta_{iw} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ as

$$\begin{aligned} \beta_{i_m, j_l}(\Delta_{i_m, j_l}) &= \begin{cases} \phi_{i, a}(\Delta_{i_m, j_l}), & 0 \leq \Delta_{i_m, j_l} < \bar{\Delta}_{i_m, j_l}, \\ \bar{\Delta}_{i_m, j_l}, & \bar{\Delta}_{i_m, j_l} \leq \Delta_{i_m, j_l}, \end{cases} \\ \beta_{ij, c}(\eta_{ij, c}) &= \begin{cases} 0, & \eta_{ij, c} < 0, \\ \phi_{i, c}(\eta_{ij, c}), & 0 \leq \eta_{ij, c} < d_{\text{con}_i}^2, \\ d_{\text{con}_i}^2, & d_{\text{con}_i}^2 \leq \eta_{ij, c}, \end{cases} \\ \beta_{iw}(\|p_{B_i}\|^2) &= (r_w - \bar{d}_{B_i})^2 - \|p_{B_i}\|^2, \end{aligned}$$

where $\bar{\Delta}_{i_m, j_l}$ is a constant satisfying $0 < \bar{\Delta}_{i_m, j_l} \leq \tilde{\Delta}_{i_m, j_l}, \forall m \in \mathcal{L}_i, l \in \mathcal{L}_j, i, j \in \mathcal{V}, i \neq j$, and $\phi_{i, a}, \phi_{i, c}$ are *strictly increasing* polynomials appropriately selected to guarantee that the functions β_{i_m, j_l} and $\beta_{ij, c}$, respectively, are twice continuously differentiable everywhere, with $\phi_{i, a}(0) = \phi_{i, c}(0) = 0, \forall i \in \mathcal{V}$. Note that the functions defined above use only local information in the sensing range d_{con_i} of agent i . The function β_{i_m, j_l} becomes zero when ellipsoid \mathcal{E}_{i_m} collides with ellipsoid \mathcal{E}_{j_l} , whereas $\beta_{ij, c}$ becomes zero when agent i loses connectivity with agent j . Similarly, β_{iw} encodes the collision of agent i with the workspace boundary.

Finally, we choose the function $\beta_i : \mathbb{R}^{Nn} \rightarrow \mathbb{R}_{\geq 0}$ as

$$\beta_i(q) = (\det(J_i(q)))^2 \beta_{iw}(\|p_{B_i}\|^2) \prod_{j \in \tilde{\mathcal{N}}_i} \beta_{ij, c}(\eta_{ij, c}) \prod_{(m, k) \in \tilde{\mathcal{L}}} \beta_{i_m, j_l}(\Delta_{i_m, j_l}) \prod_{(m, k) \in \tilde{\mathcal{L}}} \Delta_{i_m, \pi_k}(q_i), \quad (9)$$

$\forall i \in \mathcal{V}$, where $\tilde{\mathcal{T}} = \mathcal{L}_i \times \mathcal{V} \times \mathcal{L}_j, \tilde{\mathcal{L}} = \mathcal{L}_i \times (\mathcal{K} \setminus \{k_i, k'_i\})$, and we have omitted the dependence on q for brevity.

Note that we have included the term $(\det(J_i))^2$ to also account for singularities of $J_i, \forall i \in \mathcal{V}$ and the term $\prod_{(m, j, l) \in \tilde{\mathcal{T}}} \beta_{i_m, j_l}(\Delta_{i_m, j_l})$ takes into account also the collisions between the ellipsoidal rigid bodies of agent i .

We are now ready to state the main theorem, that summarizes the main results of this work.

Theorem 1: There exist positive and finite \underline{k}_i , such that, if $k_i > \underline{k}_i, \forall i \in \mathcal{V}$, and under the Assumptions 1-3, the decentralized control laws $\tau_i : \mathbb{R}^{Nn} \times \mathbb{R}^{Nn} \rightarrow \mathbb{R}^n$, with

$$\begin{aligned} \tau_i(q, \dot{q}) &= g_i(q_i) + \alpha(\varphi_i) \nabla_{q_i} \varphi_i(q) - \hat{c}_i(q_i, \dot{q}_i) \|\dot{q}_i\| \dot{q}_i \\ &\quad - \alpha(\varphi_i) \frac{\rho_i \dot{q}_i}{\tanh(\|\dot{q}_i\|^2)} \left| \sum_{j \in \mathcal{N}_i(t)} [\nabla_{q_j} \varphi_i(q)]^\top \dot{q}_j \right| - \lambda_i \dot{q}_i, \quad (10) \end{aligned}$$

$\forall i \in \mathcal{V}$, where $\alpha : [0, 1] \rightarrow [1, \infty)$, with $\alpha(x) = \frac{1}{1-x}, \lambda_i \in \mathbb{R}_{\geq 0}, \rho_i \geq 1$, along with the adaptation laws $\hat{c}_i : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$:

$$\dot{\hat{c}}_i(q_i, \dot{q}_i) = \sigma_i \|\dot{q}_i\|^2 \|\dot{q}_i\|, \quad (11)$$

with $\hat{c}_i(q_i(t_0), \dot{q}_i(t_0)) < \infty, \sigma_i \in \mathbb{R}_{\geq 0}, \forall i \in \mathcal{V}$, guarantee the transitions $(\pi_{k_i}, t_0) \xrightarrow{i} (\pi_{k'_i}, t_{f_i})$ for finite $t_{f_i}, i \in \mathcal{V}$, while ensuring $\beta_i > 0, \forall i \in \mathcal{V}$, as well as the boundedness of all closed loop signals, providing, therefore, a solution to Problem 1.

Proof: We first define the sets that represent the undesired and desired agent configurations, respectively:

$$\begin{aligned} \hat{Q}_i &= \{q \in \mathbb{R}^{Nn} \text{ s.t. } \gamma_i(q_i) > (r_{k'_i} - 3\bar{d}_{B_i})^2 \geq 1\}, \\ \tilde{Q}_i &= \mathbb{R}^{Nn} \setminus \hat{Q}_i = \{q \in \mathbb{R}^{Nn} \text{ s.t. } \gamma_i(q_i) \leq (r_{k'_i} - 3\bar{d}_{B_i})^2\}. \end{aligned}$$

The closed loop system of (2) is written as:

$$\begin{aligned} M_i(q_i) \ddot{q}_i + N_i(q_i, \dot{q}_i) \dot{q}_i + f_i(q_i, \dot{q}_i) &= \alpha(\varphi_i) \nabla_{q_i} \varphi_i(q_i) - \lambda_i \dot{q}_i \\ &\quad - \hat{c}_i(q_i, \dot{q}_i) \|\dot{q}_i\| \dot{q}_i - \alpha(\varphi_i) \frac{\rho_i \dot{q}_i}{\tanh(\|\dot{q}_i\|^2)} \left| \sum_{j \in \mathcal{N}_i(t)} [\nabla_{q_j} \varphi_i(q)]^\top \dot{q}_j \right|, \quad (12) \end{aligned}$$

$\forall i \in \mathcal{V}$. Due to Assumption 3, the domain where the functions $\alpha(\varphi_i)$ are well-defined (i.e., where $\varphi_i < 1 \Leftrightarrow \beta_i > 0$) is connected. Hence, consider the Lyapunov-like function $V : [0, 1]^N \times \mathbb{R}^{Nn} \times \mathbb{R}^N \times \mathbb{R}^{Nn} \rightarrow \mathbb{R}_{\geq 0}$, with

$$V(\varphi, \dot{q}, \tilde{c}, q) = \sum_{i \in \mathcal{V}} \ln(\alpha(\varphi_i)) + \frac{1}{2} [\dot{q}_i^\top M_i(q_i) \dot{q}_i + \frac{1}{\sigma_i} \tilde{c}_i^2]$$

where φ and \tilde{c} are the stack vectors containing all φ_i and \tilde{c}_i , respectively, $i \in \mathcal{V}$, and $\tilde{c}_i : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$, with $\tilde{c}_i(q_i, \dot{q}_i) = \hat{c}_i(q_i, \dot{q}_i) - c_i, \forall i \in \mathcal{V}$. Note that, since $\mathcal{A}_i(t_0) \in \pi_{k_i}, \mathcal{A}_i(t_0) \neq \mathcal{A}_j(t_0), \det(J_i(q_i(t_0))) \neq 0, \forall i \in \mathcal{V}$, as well as $\mathcal{E}_{i_m}(q_i(t_0)) \cap \mathcal{E}_{i_\ell}(q_i(t_0)) = \emptyset, \forall i \in \mathcal{V}, j \in \mathcal{V}, i \neq j, m, \ell \in \mathcal{L}_i, m \neq \ell$, the functions $\beta_i(q), i \in \mathcal{V}$, are strictly positive at t_0 which implies the boundedness of V at t_0 . Therefore, since $\dot{q}_i(t_0) < \infty$ and $\hat{c}_i(t_0) < \infty, \forall i \in \mathcal{V}$, there exists a positive and finite constant $1 < M < \infty$ such that $V_0 := V(\varphi(q(t_0)), \dot{q}(t_0), \tilde{c}(q(t_0), \dot{q}(t_0)), q(t_0)) \leq M$.

By differentiating V with respect to time, substituting the dynamics (2), and employing the skew symmetry of $M_i -$

$2N_i$, we obtain

$$\dot{V} = \sum_{i \in \mathcal{V}} \left\{ \dot{q}_i^\top (-\alpha(\varphi_i) \nabla_{q_i} \varphi_i + \tau_i - g_i) - \dot{q}_i^\top f_i + \frac{1}{\sigma_i} \tilde{c}_i \dot{c}_i - \alpha(\varphi_i) \sum_{j \in \mathcal{N}_i(t)} \{(\nabla_{q_j} \varphi_i)^\top \dot{q}_j\} \right\},$$

which, by substituting the control and adaptation laws (10), (11) becomes:

$$\begin{aligned} \dot{V} &= \sum_{i \in \mathcal{V}} \left\{ -\lambda_i \|\dot{q}_i\|^2 - \hat{c}_i \|\dot{q}_i\|^2 \|q_i\| - \dot{q}_i^\top f_i + \tilde{c}_i \|\dot{q}_i\|^2 \|q_i\| \right. \\ &\quad \left. - \alpha(\varphi_i) \frac{\rho_i \|\dot{q}_i\|^2}{\tanh(\|\dot{q}_i\|^2)} \left| \sum_{j \in \mathcal{N}_i(t)} \{(\nabla_{q_j} \varphi_i)^\top \dot{q}_j\} \right| \right. \\ &\quad \left. - \alpha(\varphi_i) \sum_{j \in \mathcal{N}_i(t)} \{(\nabla_{q_j} \varphi_i)^\top \dot{q}_j\} \right\} \\ &\leq \sum_{i \in \mathcal{V}} \left\{ -\lambda_i \|\dot{q}_i\|^2 - (\hat{c}_i - c_i - \tilde{c}_i) \|\dot{q}_i\|^2 \|q_i\| \right. \\ &\quad \left. - \alpha(\varphi_i) \left| \sum_{j \in \mathcal{N}_i(t)} \{(\nabla_{q_j} \varphi_i)^\top \dot{q}_j\} \right| \left(\frac{\rho_i \|\dot{q}_i\|^2}{\tanh(\|\dot{q}_i\|^2)} - 1 \right) \right\} \end{aligned}$$

where we have used the property $\|f_i(q_i, \dot{q}_i)\| \leq c_i \|q_i\| \|\dot{q}_i\|$. Since $\tilde{c}_i = \hat{c}_i - c_i$ and $\frac{\rho_i \|\dot{q}_i\|^2}{\tanh(\|\dot{q}_i\|^2)} \geq 1, \forall \rho_i \geq 1, \dot{q}_i \in \mathbb{R}^n, i \in \mathcal{V}$, we obtain $\dot{V} \leq -\sum_{i \in \mathcal{V}} \lambda_i \|\dot{q}_i\|^2$, which implies that V is non-increasing along the trajectories of the closed loop system. Hence, we conclude that $V \leq V_0 \leq M$, where M depends on the initial conditions, as well as the boundedness of $\tilde{c}_i, \varphi_i, \dot{q}_i$ and hence of $\hat{c}_i, \forall i \in \mathcal{V}, t \in \mathbb{R}_{\geq 0}$. Therefore,

$$\begin{aligned} \ln(\alpha(\varphi_i(q))) &= \ln\left(\frac{1}{1 - \varphi_i(q)}\right) \leq M \Leftrightarrow \\ \varphi_i(q) &\leq \bar{\varphi} := 1 - \frac{1}{e^M} < 1, \end{aligned} \quad (13)$$

$\forall q \in \mathbb{R}^{Nn}, i \in \mathcal{V}$, which implies that $\beta_i(q) > 0, \forall q \in \mathbb{R}^{Nn}, i \in \mathcal{V}$. Hence, inter-agent collisions, collision with the undesired regions and the obstacle boundary, connectivity losses between the subsets of the initially connected agents and singularity configurations are avoided. Moreover, by invoking LaSalle's Invariance Principle, the system converges to the largest invariant set contained in

$$S = \{(q, \dot{q}) \in \mathbb{R}^{Nn} \times \mathbb{R}^{Nn} \text{ s.t. } \dot{q} = 0_{n \times 1}\}. \quad (14)$$

For S to be invariant, we require that $\ddot{q}_i = 0_{n \times 1}, \forall i \in \mathcal{V}$, and thus we conclude for the closed loop system (12) that $\nabla_{q_i} \varphi_i(q) = 0_{n \times 1}, \forall i \in \mathcal{V}$, since $f_i(q_i, 0_{n \times 1}) = 0_{n \times 1}, \forall q_i \in \mathbb{R}^n$, in view of Assumption 1. Therefore, each agent $i \in \mathcal{V}$ will converge to a critical point of φ_i , i.e., all the configurations where $\nabla_{q_i} \varphi_i(q) = 0_{n \times 1}, \forall i \in \mathcal{V}$. It was proved in [23] that the critical points of φ_i in (7) are identical with the ones of $\frac{\gamma_i^{k_i}}{\beta_i}$. Hence, each agent $i \in \mathcal{V}$ will converge to the set where $\nabla_{q_i} \frac{\gamma_i^{k_i}}{\beta_i} = 0$, i.e.,

$$k_i \beta_i \gamma_i^{k_i-1} \nabla_{q_i} \gamma_i = \gamma_i^{k_i} \nabla_{q_i} \beta_i \Leftrightarrow k_i \beta_i \nabla_{q_i} \gamma_i = \gamma_i \nabla_{q_i} \beta_i. \quad (15)$$

Note that (15) holds for all $q_i \in \{q_i \in \mathbb{R}^n \text{ s.t. } \gamma_i(q_i) = 0\}$, since it can be concluded from (8) that $\gamma_i(q_i) = 0 \Leftrightarrow \nabla_{q_i} \gamma_i(q_i) = 0, \forall i \in \mathcal{V}$. Next, we must ensure that (15) does not hold when agent i is not in region $\pi_{k'_i}$ in order for the system not to stop at an undesired local minimum. More specifically, we design k_i such that (15) never holds in \hat{Q}_i and hence, q_i ends up in the set $\tilde{Q}_i = \mathbb{R}^{Nn} \setminus \hat{Q}_i, \forall i \in \mathcal{V}$. By taking norms in (15), we obtain

$$k_i \beta_i \|\nabla_{q_i} \gamma_i\| = \gamma_i \|\nabla_{q_i} \beta_i\|. \quad (16)$$

In view of (8), it holds that $\|\nabla_{q_i} \gamma_i(q_i)\| = 2(\gamma_i(q_i))^{\frac{1}{2}}, \forall q_i \in \mathbb{R}^n$. Hence, in order for (16) to hold when $q \in \hat{Q}_i$, where $\gamma_i(q_i) > (r_{k'_i} - 3\bar{d}_{B_i})^2$, it must hold that $k_i \beta_i = \frac{1}{2} \sqrt{\gamma_i} \|\nabla_{q_i} \beta_i\|$, for some $q \in \hat{Q}_i$. Hence, a sufficient condition for (16) not to hold for any $q \in \hat{Q}_i$ is

$$k_i > \sup_{q \in \hat{Q}_i} \left\{ \frac{1}{2\beta_i} \sqrt{\gamma_i} \|\nabla_{q_i} \beta_i\| \right\}, \quad (17)$$

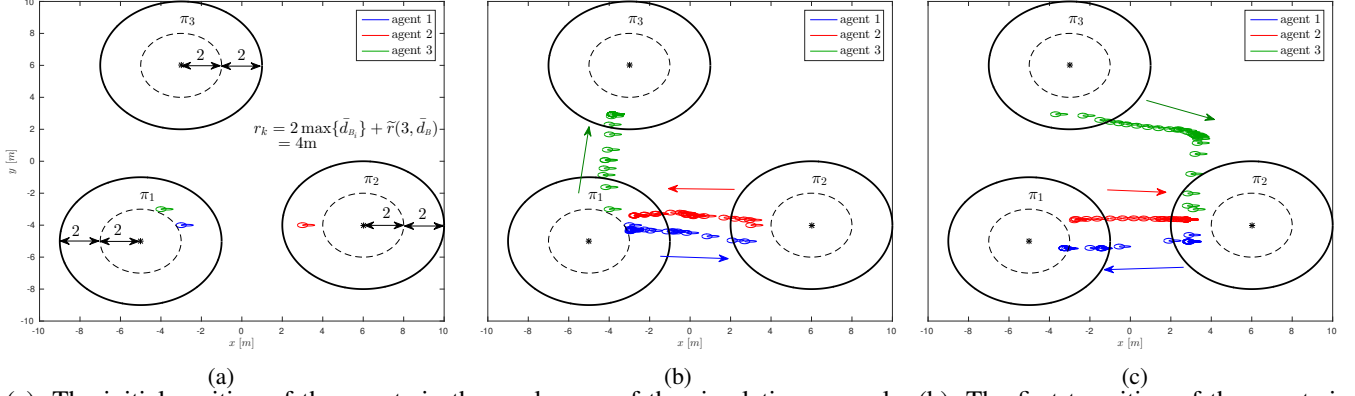
$\forall i \in \mathcal{V}$. From (13), by substituting (7), we obtain that

$$\begin{aligned} \varphi_i(q) \leq \bar{\varphi} < 1 &\Leftrightarrow \beta_i(q) \geq \gamma_i^{k_i}(q) \left(\frac{1}{\bar{\varphi}^{k_i}} - 1 \right) \Leftrightarrow \\ \beta_i(q) &\geq \gamma_i^{k_i}(q) \left(\frac{1}{\bar{\varphi}} - 1 \right), \end{aligned} \quad (18)$$

$\forall q \in \mathbb{R}^{Nn}, i \in \mathcal{V}$, since $\bar{\varphi} < 1$. In addition, since $q \in \hat{Q}_i$, it holds that $\gamma_i(q_i) \geq (r_{k'_i} - 3\bar{d}_{B_i})^2$, and thus $\gamma_i^{k_i}(q_i) \geq (r_{k'_i} - 3\bar{d}_{B_i})^{2k_i}$. Hence, in view of (5) and the fact that $k_i > 1, \forall i \in \mathcal{V}$, it holds that $(r_{k'_i} - 3\bar{d}_{B_i})^{2k_i} \geq 1, \forall i \in \mathcal{V}$. Therefore, (18) becomes $\beta_i(q) \geq \beta := (\frac{1}{\bar{\varphi}} - 1) > 0, \forall q \in \hat{Q}_i, i \in \mathcal{V}$, and hence, (17) is implied by

$$k_i > \underline{k}_i := \frac{1}{2\beta} \sup_{q \in \hat{Q}_i} \{\sqrt{\gamma_i} \|\nabla_{q_i} \beta_i\|\}, \forall i \in \mathcal{V}. \quad (19)$$

Hence, by designing k_i as in (19), we guarantee that $\varphi_i(q)$ satisfies $\nabla_{q_i} \varphi_i(q) \neq 0_{n \times 1}, \forall q \in \hat{Q}_i$. Hence, each agent will converge to the configuration set $\{q \in \hat{Q}_i = \mathbb{R}^{Nn} \setminus \tilde{Q}_i \text{ s.t. } \nabla_{q_i} \varphi_i(q) = 0_{n \times 1}\}, \forall i \in \mathcal{V}$, which are all the configurations such that $\gamma_i(q_i) \leq (r_{k'_i} - 3\bar{d}_{B_i})^2$, i.e. $\lim_{t \rightarrow \infty} \gamma_i(q_i(t)) \leq (r_{k'_i} - 3\bar{d}_{B_i})^2$. Therefore, we can conclude that there exist finite constants $t_{f_i}, i \in \mathcal{V}$, such that $\gamma_i(q_i(t_{f_i})) \leq (r_{k'_i} - \bar{d}_{B_i})^2, \forall i \in \mathcal{V}$ (see Fig. 2), and hence, each agent i will be at its goal region $p_{k'_i}$ at time $t_{f_i}, \forall i \in \mathcal{V}$. Moreover, since $\beta_i(q) > 0, \forall q \in \mathbb{R}^{Nn}, \forall i \in \mathcal{V}$, the terms $\nabla_{q_i} \gamma_i, \nabla_{q_i} \beta_i$ remain always bounded and thus $\nabla_{q_i} \varphi_i(q)$ remains bounded for all $t \in \mathbb{R}_{\geq 0}, i \in \mathcal{V}$. In addition, the boundedness of q_i, \dot{q}_i implies the boundedness of the adaptation laws $\hat{c}_i, \forall i \in \mathcal{V}$. Hence, in order to prove the boundedness of the control laws (10), we need to prove the boundedness of the terms $\frac{\rho_i \dot{q}_i}{\tanh(\|\dot{q}_i\|^2)} \left| \sum_{j \in \mathcal{N}_i(t)} [\nabla_{q_j} \varphi_i(q)]^\top \dot{q}_j \right|, \forall i \in \mathcal{V}$, i.e. we need to prove that there does not exist a time instant t^* such that $\mathcal{N}_i(t^*) \neq \emptyset, \dot{q}_i(t^*) = 0_{n \times 1}$ and $\dot{q}_j(t^*) \neq 0_{n \times 1}$, for some $j \in \mathcal{N}_i(t^*)$. By contradiction, assume, that such a time instant exists, i.e. $\exists t^* \in \mathbb{R}_{\geq 0}$ such that $\mathcal{N}_i(t^*) \neq \emptyset, \dot{q}_i(t^*) = 0_{n \times 1}$ and $\dot{q}_j(t^*) \neq 0_{n \times 1}$, for some $j \in \mathcal{N}_i(t^*)$. Then, $\tau_i(q(t^*), \dot{q}(t^*)) = \infty$, and hence, $\dot{q}_i(t^* + \varepsilon_t) \neq 0_{n \times 1}$, where ε_t is an arbitrarily small positive constant. This, however, is a contradiction, since the set (14)



(a): The initial position of the agents in the workspace of the simulation example. (b): The first transition of the agents in the workspace. Agent 1 transits from π_1 to π_2 , agent 2 from π_2 to π_1 , and agent 3 from π_1 to π_3 . (c): The second transition of the agents in the workspace. Agent 1 transits from π_2 to π_1 , agent 2 from π_1 to π_2 , and agent 3 from π_3 to π_2 .

is invariant. We also conclude, therefore, the boundedness of the control protocol (10). ■

Remark 1: Consider the special case that a subset of agents $\mathcal{V}_s \subseteq \mathcal{V}$ have the same goal region and one of them (let agent $z \in \mathcal{V}_s$) ends up in the configuration where $\gamma_z(q_z) = 0$ (i.e., the center of the region). Then, it follows from (18) that $\beta_z(q) \geq 0$ at this configuration, and since $\dot{q}_z = 0$, the specific agent will no longer try to avoid collisions. In this case, however, the terms of β_z that might vanish concern exclusively collisions with the remaining agents $i \in \mathcal{V}_s \setminus \{z\}$ that have the same goal region. These agents then, which are not in the special configuration $\gamma_i(q_i) = 0, \forall i \in \mathcal{V}_s \setminus \{z\}$ (since that would imply collision with agent z), will guarantee that the “dangerous” terms of β_z will remain strictly positive due to (18), since these terms belong also to the obstacle functions $\beta_i, i \in \mathcal{V}_s \setminus \{z\}$.

Remark 2: Note that the design of the obstacle functions (9) renders the control laws (10) decentralized, in the sense that each agent uses only local information with respect to its neighboring agents, according to its limited sensing radius. Moreover, the potential functions φ_i , which constitute the basic tool for the proposed methodology, are not correct-by-construction navigation functions, despite the potential similarities of the gain tuning for the manipulation of the critical points. Therefore, mainly due to the 2nd order systems that are considered in this work, the tedious procedure of proving that a designed potential field is a navigation function, which was introduced in [23] and is followed in many works, is avoided, simplifying thus the corresponding technical details. Finally, note that the considered dynamic model (2) applies for more general manipulation robots (e.g. underwater or aerial manipulators), without limiting the proposed methodology to mobile ones.

B. Hybrid Control Framework

Due to the proposed continuous control protocol of Section IV-A, the transitions $(\pi_{k_i}, t_0) \xrightarrow{i} (\pi_{k'_i}, t_{f_i})$ of Problem 1 are well-defined, according to Def. 3. Moreover, since all the agents $i \in \mathcal{V}$ remain connected with the subset of their initial neighbors $\tilde{\mathcal{N}}_i$ and there exist finite constants t_{f_i} , such

that $\mathcal{A}_i(q_i(t_{f_i})) \in \pi_{k'_i}, \forall i \in \mathcal{V}$, all the agents are aware of their neighbors state, when a transition is performed. Hence, the transition system (6) is well defined, $\forall i \in \mathcal{V}$. Consider, therefore, that $\mathcal{A}_i(q_i(0)) \in \pi_{k_{i,0}}, k_{i,0} \in \mathcal{K}, \forall i \in \mathcal{V}$, as well as a given desired path for each agent, that does not violate the connectivity condition of Problem 1. Then, the iterative application of the control protocol (10) for each transition of the desired path of agent i guarantees the successful execution of the desired paths, with all the closed loop signals being bounded.

Remark 3: Note that, the proposed methodology is also valid in the case where only a subset of all the agents are in regions of interest at t_0 . Hence, if the members of an initially connected subset of neighbors reach their destinations, they can start directly with their next transitions, without having to wait for the remaining agents, which will be still performing their corresponding transitions.

V. SIMULATION RESULTS

To demonstrate the validity of the proposed methodology, we consider the simplified example of three agents in a workspace with $r_0 = 12\text{m}$. Each agent consists of a mobile base and a rigid link connected with a rotational joint, with $\bar{d}_{B_i} = 1\text{m}, \forall i \in \{1, 2, 3\}$. Then, it holds for the minimum region radius $1 < \tilde{r}(N, \bar{d}_B) \leq 2\text{m}$, and hence, from (4), we obtain $\tilde{r}(N, \bar{d}_B) = 2\text{m}$. Hence, the smallest region radius in order for Assumption 3-(i) to hold is $r_k = 2 \max_{i \in \{1, 2, 3\}} \{2\bar{d}_{B_i}\} + \tilde{r}(N, \bar{d}_B) = 4\text{m}$. We also choose $p_1 = [-5, -5]\text{m}, p_2 = [6, -4]\text{m}, p_3 = [-3, 6]\text{m}$, which ensures that Assumption 3-(ii) also holds. The initial base positions are taken as $p_{B_1} = [-3, -4]^\top\text{m}, p_{B_2} = [3, -4]^\top\text{m}, p_{B_3} = [-4, -5]^\top\text{m}$ with $\bar{d}_{B_i} = 1.25\text{m}, \forall i \in \{1, 2, 3\}$, which imply that $\mathcal{A}_1(q_1(0)), \mathcal{A}_3(q_3(0)) \in \pi_1$ and $\mathcal{A}_2(q_2(0)) \in \pi_2$ (see Fig. 3a). The control inputs for the agents are the 2D force acting on the mobile base, and the joint torque of the link. We also consider a sensing radius of $d_{\text{con}_i} = 8\text{m}$ and the subsets of initial neighbors as $\tilde{\mathcal{N}}_1 = \{2\}, \tilde{\mathcal{N}}_2 = \{1, 3\}$, and $\tilde{\mathcal{N}}_3 = \{2\}$, i.e., agent 1 has to stay connected with agent 2, agent 2 has to stay connected with agents 1 and 3 and agent 3 has to stay connected with agent 2.

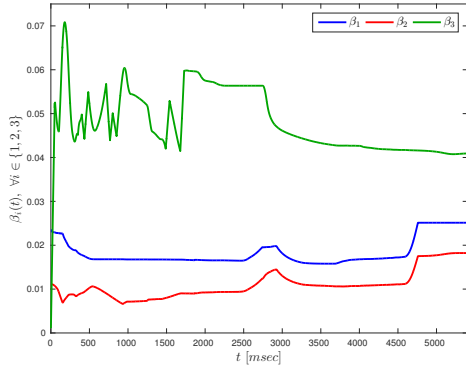


Fig. 4: The obstacle functions $\beta_i, i \in \{1, 2, 3\}$, which remain strictly positive.

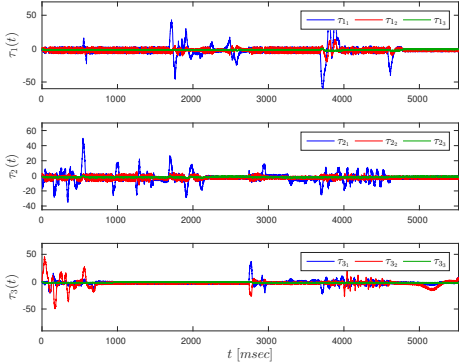


Fig. 5: The resulting control inputs $\tau_i, \forall i \in \{1, 2, 3\}$ for the two transitions.

The agents are required to perform two transitions. Regarding the first transition, we choose $\pi_{k'_1} = \pi_2$ for agent 1, $\pi_{k'_2} = \pi_1$ for agent 2, and $\pi_{k'_3} = \pi_3$, for agent 3. Regarding the second transition, we choose $\pi_{k'_1} = 1, \pi_{k'_2} = 2$, and $\pi_{k'_3} = 2$. The control parameters and gains where chosen as $k_i = 5, \lambda_i = 10, \rho_i = 1$, and $\sigma_i = 0.01, \forall i \in \{1, 2, 3\}$. The simulation results are depicted in Fig. 3b-6. In particular, Fig. 3b and 3c illustrate the two consecutive transitions of the agents. Fig. 4 depicts the obstacle functions β_i which are strictly positive, $\forall i \in \{1, 2, 3\}$. Finally, the control inputs are given in Fig. 5 and the parameter errors \tilde{c} are shown in Fig. 6, which indicates their boundedness. As proven in the theoretical analysis, the transitions are successfully performed while satisfying all the desired specifications.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper we designed decentralized abstractions for multiple mobile manipulators by guaranteeing the navigation of the agents among predefined regions of interest, while guaranteeing inter-agent collision avoidance and connectivity maintenance for the initially connected agents. We proposed a novel approach for ellipsoid collision avoidance as well as appropriately chosen potential functions that are free of undesired local minima. Future efforts will be devoted towards addressing abstractions of cooperative tasks between the agents by employing hybrid control techniques as well as development of tuning-free methodologies.

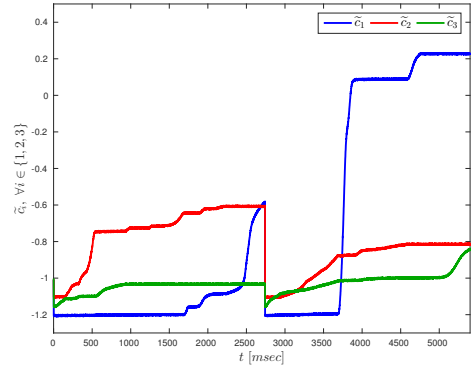


Fig. 6: The parameter deviations $\tilde{c}_i, \forall i \in \{1, 2, 3\}$, which are shown to be bounded.

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